Introduction into integrable quantum field theories

Abstract

The lecture course is dedicated to two-dimensional integrable field theories, including the following topics: conformal field theory, massive integrable models, bootstrap methods.

1. Free field theories
   (a) Massless free boson on the cylinder and on the plane
   Massless free boson on the cylinder. Conformal map to the plane. Wick's theorem and exponential operators. Energy-momentum tensor and Virasoro algebra.
   (b) Massless free fermion on the cylinder and on the plane
   Massless free Majorana fermion on the cylinder. Neveu–Schwarz and Ramond boundary conditions. Conformal map to the plane. Order and disorder operators. Energy-momentum tensor and Virasoro algebra.
   (c) Massive free fields.
   Massive fields. Nonequivalence of radial and plane quantizations. Energy-momentum conservation law.
2. Conformal field theory
   (a) Energy-momentum tensor and Virasoro algebra.
   Operator product expansion for the energy-momentum tensor. Primary and descendant operators. Virasoro algebra and its representations. Null vectors and degenerate modules.
   (b) Conformal blocks and crossing symmetry
   Operator product expansion of two primary operators. Fusion rules. Structure constants and conformal families. Four-point correlation functions and conformal blocks. Minimal conformal models.
   (c) Free boson representation.
   Charge at infinity and modified energy-momentum tensor. Representation of primary fields with exponential operators. Representation for conformal blocks and correlation functions.
3. Boson-fermion correspondence
   (a) $O(2)$ model and sine-Gordon model.
   Vortices in the $O(2)$ model and their description in terms of the dual field. Action of vortices. Condensation of vortices and Berezinsky–Kosterlitz–Thouless transition. Relation to the sine-Gordon model.
   (b) Thirring model and its bosonization.
   Massive and massless Thirring models. Bosonization of the massless model. Massive term as a cosine perturbation.
   (c) Bethe Ansatz for the Thirring model.
   Pseudovacuum in the Thirring model. Pseudoparticles and their scattering. Construction of the true vacuum. Hole and its mass.
4. Exact S-matrices
   (a) Perturbation theory in the Thirring model.
   Two-particle scattering in the Thirring model. Cancellation of the multiple production amplitudes. The role of integrals of motion.
   (b) Exact S matrices and Yang–/Baxter equation.
   Factorized scattering and the Yang–/Baxter equation. Crossing symmetry and unitarity. A solution to the Yang–/Baxter equation. Its connection to the Thirring and sineGordon theory.
   (c) Questions, discussion and examination.