Lecture 4
Free field representation: SOS model

The bosonization or free field representation appeared in conformal field theory in the works by Feigin and Fuchs and by Dotsenko and Fateev in 1983. It is no use to recall these papers for our purposes. The most important outcome of these papers for us is that some linear equations can be solved by representing the solution in terms of expectation values of some quantum operators. The trace form of the functions we want to obtain prompt us that it must be a thermal average. If the Hamiltonian H is quadratic in the bosonic field and the operators $\Phi(u)_n^{n'}$ is expressed as exponentials of this field, the problem will be solvable.

Let me first formally introduce the construction by Lukyanov and Pugai (1996) and then to explain how it can be obtained.

Consider a Heisenberg algebra of operators a_k ($k \in \mathbb{Z} \setminus \{0\}$) and a pair of 'zero-mode' operators $\mathcal{P}$ and $\mathcal{Q}$ with the commutation relations

$$[\mathcal{P}, \mathcal{Q}] = -\mathrm{i}, \qquad [a_k, a_l] = k\frac{[\![k]\!]_x [\![(r-1)k]\!]_x}{[\![2k]\!]_x [\![rk]\!]_x}\delta_{k+l,0} \quad \text{with} \quad [\![u]\!]_x = \frac{x^u - x^{-u}}{x - x^{-1}}. \tag{1}$$

The 'q-number' $[\![u]\!]_x$ here should not be confused with the 'elliptic q-numbers' $[u]_i$ ($i = 1, \ldots, 4$). It is also useful to introduce the operators

$$\tilde{a}_k = \frac{[\![rk]\!]_x}{[\![(r-1)k]\!]_x} a_k. \tag{2}$$

The normal ordering operation $:\ldots:$ places $\mathcal{P}$ to the right of $\mathcal{Q}$ and a_k with positive k to the right of a_{-k}. It will be convenient to assign

$$\alpha_+ = \sqrt{a_+} = \sqrt{\frac{r}{r-1}}, \qquad \alpha_- = -\sqrt{a_-} = -\sqrt{\frac{r-1}{r}}, \qquad 2\alpha_0 = \alpha_+ + \alpha_- = \frac{1}{\sqrt{r(r-1)}}. \tag{3}$$

Now introduce the fields

$$\begin{aligned} \varphi(z) &= \frac{\alpha_-}{\sqrt{2}}(\mathcal{Q} - \mathrm{i}\mathcal{P}\log z) - \sum_{k \neq 0} \frac{a_k}{\mathrm{i}k} z^{-k}. \\ \tilde{\varphi}(z) &= \frac{\alpha_+}{\sqrt{2}}(\mathcal{Q} - \mathrm{i}\mathcal{P}\log z) + \sum_{k \neq 0} \frac{\tilde{a}_k}{\mathrm{i}k} z^{-k}. \end{aligned} \tag{4}$$

These fields enter the exponential operators

$$\begin{aligned} V(u) &= z^{(r-1)/4r} :e^{\mathrm{i}\varphi(z)}:, \qquad & \bar{V}(u) &= z^{(r-1)/r} :e^{-\mathrm{i}\varphi(x^{-1}z) - \mathrm{i}\varphi(xz)}:, \\ \tilde{V}(u) &= z^{r/4(r-1)} :e^{\mathrm{i}\tilde{\varphi}(z)}:, \qquad & \tilde{\bar{V}}(u) &= z^{r/(r-1)} :e^{-\mathrm{i}\tilde{\varphi}(x^{-1}z) - \mathrm{i}\tilde{\varphi}(xz)}:, \end{aligned} \tag{5}$$

and Lukyanov's screening operators

$$\begin{aligned} x(u, C) &= \frac{\epsilon}{\eta} \int_C \frac{dv}{\mathrm{i}\pi} \bar{V}(v) \frac{[v - u + \frac{1}{2} - \sqrt{2r(r-1)}\,\mathcal{P}]}{[v - u - \frac{1}{2}]}, \\ \tilde{x}(u, C) &= \frac{\epsilon}{\eta'} \int_C \frac{dv}{\mathrm{i}\pi} \tilde{\bar{V}}(v) \frac{[v - u - \frac{1}{2} + \sqrt{2r(r-1)}\,\mathcal{P}]'}{[v - u + \frac{1}{2}]'}. \end{aligned} \tag{6}$$

The constants η, η' will be fixed as

$$\begin{aligned} \eta^{-1} &= \mathrm{i}[1]\, x^{\frac{r-1}{2r}} \frac{(x^2; x^{2r})_\infty}{(x^{2r-2}; x^{2r})_\infty} \frac{(x^6; x^4, x^{2r})_\infty (x^{2r+2}; x^4, x^{2r})_\infty}{(x^4; x^4, x^{2r})_\infty (x^{2r+4}; x^4, x^{2r})_\infty}, \\ \eta'^{-1} &= -\frac{2\epsilon}{\pi}[1]' x^{-\frac{r}{2(r-1)}} \frac{(x^{2r-2}; x^{2r-2})^2_\infty}{(x^{2r}; x^{2r-2})^2_\infty} \frac{(x^4; x^4, x^{2r-2})_\infty (x^{2r+2}; x^4, x^{2r-2})_\infty}{(x^2; x^4, x^{2r-2})_\infty (x^{2r+4}; x^4, x^{2r-2})_\infty}. \end{aligned} \tag{7}$$

Now let us fix the contours. Let C_u^- and C_u^+ go from $u - \frac{\mathrm{i}\pi}{2\epsilon}$ to $u + \frac{\mathrm{i}\pi}{2\epsilon}$ to the left and to the right of u respectively.

(We assume that the contours $C_u^{\pm}$ go to the left of all poles in the 'main rectangle' related to the operators that are to the right of the screening operator and to the right of all poles related to the operators placed to the left of the screening operators. The 'main rectangle' is understood as a rectangle with sides r along the real axis and $\frac{\pi}{\epsilon}$ along the imaginary axis that contains all points u_i, v_i *etc.* It is well defined for large enough r and for points u_i, v_i,... close enough to each other. In the general case the operator products are considered as analytic continuation from this region.)

Then

$$\begin{aligned} X(u) &= x(u, C^-_{u+1/2}), & Y(u) &= x(u-1, C^+_{u-1/2}), \\ \tilde{X}(u) &= \tilde{x}(u, C^-_{u-1/2}), & \tilde{Y}(u) &= \tilde{x}(u+1, C^+_{u+1/2}). \end{aligned} \tag{8}$$

These operators satisfy the equations

$$Y(u)V(u) = V(u)X(u), \qquad \tilde{Y}(u)\tilde{V}(u) = \tilde{V}(u)\tilde{X}(u). \tag{9}$$

Define the Fock spaces $\mathcal{F}_{mn}$ generated by the operators a_{-k} $(k > 0)$ from the highest weight vectors $|P_{mn}\rangle$ such that

$$a_k|P_{mn}\rangle = 0 \quad (k > 0), \qquad \mathcal{P}|P_{mn}\rangle = P_{mn}|P_{mn}\rangle, \qquad P_{mn} = \frac{1}{\sqrt{2}}(\alpha_+ m + \alpha_- n). \tag{10}$$

There are strong evidences that $\mathcal{F}_{mn}$ can be identified with $\mathcal{H}_{mn}$ for generic r.

The vertex operators are defined on $\mathcal{F}_{mn}$ as follows:

$$\begin{aligned} \Phi(u)_n^{n+1} &= \frac{\mathrm{i}^{m-n}}{[n]} V(u), \\ \Phi(u)_n^{n-1} &= -\frac{\mathrm{i}^{m-n}}{[n]} V(u)X(u), \\ \Psi^*(u)_m^{m+1} &= \tilde{V}(u), \\ \Psi^*(u)_m^{m-1} &= (-1)^{m-n}\tilde{Y}(u)\tilde{V}(u). \end{aligned} \tag{11}$$

The corner Hamiltonian H_{mn} is the restriction to $\mathcal{F}_{mn}$ of the operator

$$H = \frac{\mathcal{P}^2}{2} + \sum_{k=1}^{\infty} \frac{[\![2k]\!]_x [\![rk]\!]_x}{[\![k]\!]_x [\![(r-1)k]\!]_x} a_{-k} a_k. \tag{12}$$

Ooh! This is the end at last!

The operators H and $\Phi(u)_n^{n'}$ satisfy the necessary algebra of commutation relations. Besides, the operators $\Psi^*(u)_n^{n'}$ satisfy a similar algebra

$$\Psi^*(u)_m^{m'} x^{2vH_{mn}} = x^{2vH_{m'n}} \Psi^*(u-v)_m^{m'}, \tag{13}$$

$$\sum_{s'} \tilde{W}\left[\begin{matrix} m' & s \\ s' & m \end{matrix}\middle| u_1 - u_2\right] \Psi^*(u_1)_{s'}^{m'} \Psi^*(u_2)_m^{s'} = \Psi^*(u_2)_s^{m'} \Psi^*(u_1)_m^s, \tag{14}$$

$$\Psi(u')_{m''}^{m'} \Psi^*(u)_m^{m''} = \frac{1}{\pi}\frac{\delta_m^{m'}}{u'-u} + O(1), \qquad \Psi(u)_m^{m'} = \frac{1}{[m]'}\Psi^*(u-1)_m^{m'}. \tag{15}$$

Here

$$\tilde{W}\left[\begin{matrix} m_4 & m_3 \\ m_1 & m_2 \end{matrix}\middle| u\right] = -\, W\left[\begin{matrix} m_4 & m_3 \\ m_1 & m_2 \end{matrix}\middle| u\right]\Bigg|_{r\to r-1}.$$

The operators $\Phi(u)$ are called the *type I* vertex operators, while $\Psi^*(u)$ are called the *type II* vertex operators. The difference between these operators is in their physical meaning. The type I operators are, as we already said, the half transfer matrices, while the type II vertex operators represent one-particle excitation states. Both types of operators satisfy the relation

$$\Phi(u_1)_n^{n'} \Psi^*(u_2)_m^{m'} = \tau(u_1 - u_2)\Psi^*(u_2)_m^{m'} \Phi(u_1)_n^{n'}, \qquad \tau(u) = \mathrm{i}\frac{\theta_1\left(\frac{1}{4} - \frac{u}{2}; \frac{\mathrm{i}\pi}{2\epsilon}\right)}{\theta_1\left(\frac{1}{4} + \frac{u}{2}; \frac{\mathrm{i}\pi}{2\epsilon}\right)}. \tag{16}$$

The function $\prod_{j=1}^{l}\tau(u-v_j)$ is the eigenvalue of the transfer matrix $T_{\mathrm{SOS}}(u)$ on the excited states of l particles. The functions $\tilde W\left[\begin{matrix}m_4 & m_3\\ m_1 & m_2\end{matrix}\middle| v_1-v_2\right]$ provide the scattering matrix of two excitations. A trace function with a product of $\Psi^*(v)$ and $\Psi(v)$ inserted represents a matrix element of a local operator described by Φs instead of its vacuum expectation value. Namely,

$$\left\langle \begin{matrix} mm'_1\ldots m'_{l'-1}m' \\ v'_1,\ldots,v'_{l'}\end{matrix}\middle| \mathcal{O}\Big(n\begin{matrix}n_1\ldots n_{k-1}\\ n'_1\ldots n'_{k-1}\end{matrix}n'\Big)\middle| \begin{matrix} mm_1\ldots m_{l-1}m' \\ v_1,\ldots,v_l\end{matrix}\right\rangle$$

$$= \frac{1}{\chi_m}\operatorname{Tr}_{\mathcal{F}_{mn}}(\Psi(v'_1)^m_{m'_1}\ldots\Psi(v'_{l'})^{m'_{l'-1}}_{m'}\Psi^*(v_l)^{m'}_{m_{l-1}}\ldots\Psi^*(v_1)^{m_1}_m$$
$$\times\Phi^*(u_1)^n_{n'_1}\ldots\Phi^*(u_k)^{n'_{k-1}}_{n'}\Phi(u_k)^{n'}_{n_{k-1}}\ldots\Phi(u_1)^{n_1}_n x^{4H}),$$

where $\mathcal{O}\Big(n\begin{matrix}n_1\ldots n_{k-1}\\ n'_1\ldots n'_{k-1}\end{matrix}n'\Big)$ is the operator corresponding to the picture in the last lecture. The corresponding vacuum expectation values are just the probabilities $P\Big(n\begin{matrix}n_1\ldots n_{k-1}\\ n'_1\ldots n'_{k-1}\end{matrix}n'\Big)$.

Now let us make some comments on the construction. Derivation of the construction above starts from the following observation. Consider first the commutation relation

$$\Phi(u_1)^{n+2}_{n+1}\Phi(u_2)^{n+1}_n = W\left[\begin{matrix}n+2 & n+1\\ n+1 & n\end{matrix}\middle| u_1-u_2\right]\Phi(u_2)^{n+2}_{n+1}\Phi(u_1)^{n+1}_n.$$

Since

$$W\left[\begin{matrix}n+2 & n+1\\ n+1 & n\end{matrix}\middle| u\right] = R_0(u) = z^{(r-1)/2r}\frac{g(z^{-1})}{g(z)},\qquad z=x^{2u},$$

we can rewrite it as

$$(z_2/z_1)^{(r-1)/4r}g^{-1}(z_2/z_1)\Phi(u_1)^{n+2}_{n+1}\Phi(u_2)^{n+1}_n = (z_1/z_2)^{(r-1)/4r}g^{-1}(z_1/z_2)\Phi(u_2)^{n+2}_{n+1}\Phi(u_1)^{n+1}_n$$

This can be reproduced if $\Phi(u)^{n+1}_n\sim{:}e^{\mathrm{i}\varphi(u)}{:}$ with $\varphi(u)$ has the form (4). The pairs (a_n,a_{-n}) are supposed to form independent Heisenberg algebra, but the normalization from (1) is not supposed. It is known that

$${:}e^{\mathrm{i}\varphi_1}{:}{:}e^{\mathrm{i}\varphi_2}{:} = e^{-\langle 0|\varphi_1\varphi_2|0\rangle}{:}e^{\mathrm{i}\varphi_1+\mathrm{i}\varphi_2}{:},$$

where φ_1, φ_2 are any linear combinations of $\mathcal{P}$, $\mathcal{Q}$, a_k. It means that if

$$\langle 0|\varphi(u_1)\varphi(u_2)|0\rangle = -\log((z_2/z_1)^{-(r-1)/4r}g(z_2/z_1)),$$

we will be able to satisfy our equation.

Let us represent $\log g(z)$ in the form of a series in z. Namely, use the identity

$$\log(z;p_1,p_2)_\infty = \sum_{n_1,n_2=0}^{\infty}\log(1-zp_1^{n_1}p_2^{n_2}) = -\sum_{n_1,n_2=0}^{\infty}\sum_{m=1}^{\infty}\frac{z^m p_1^{mn_1}p_2^{mn_2}}{m} = -\sum_{m=1}^{\infty}\frac{z^m}{(1-p_1^m)(1-p_2^m)}.$$

Applying it to the definition of $g(z)$, we obtain

$$\log g(z) = -\sum_{m=1}^{\infty}\frac{(x^{2m}+x^{(2r+2)m}-x^{4m}-x^{2rm})z^m}{(1-x^{4m})(1-x^{2rm})}.$$

This reproduces the normalizations in (1).

Obtaining the other relations, e.g.

$$\Phi(u_1)^n_{n-1}\Phi(u_2)^{n-1}_n = (\text{something})\times\Phi(u_2)^n_{n-1}\Phi(u_1)^{n-1}_n + (\text{something})\times\Phi(u_2)^n_{n+1}\Phi(u_1)^{n+1}_n$$

Without the second term it could be reproduced by pure exponentials (but with a wrong coefficient!), but the second term spoils everything. From conformal field theory it is known that such commutation relations can by obtained by use of the *screening operators*, which are integral of exponentials. The particular form of the screening operator (6) was guessed after long attempts to use a simpler form without an elliptic function of the zero mode operator. The normalization constant η is extracted from the normalization property for the vertex operators.

The Ψ operators appeared as a natural generalizations of some operators in the conformal field theory. Their meaning as representatives of excited states was established by comparing with similar construction for the six-vertex model by Jimbo and Miwa and with Lukyanov's construction for form factors in quantum field theory.

Similar operators $\Psi^*_\varepsilon(u)$ must exist for the eight-vertex model. The algebra of these operators was established by Foda, Iohara, Jimbo, Miwa, and Yan in 1994 in the study of the elliptic algebra $\mathcal{A}_{q,p}(\widehat{sl}_2)$ [1]. Let us write down the result:

$$\sum_{\varepsilon'_1\varepsilon'_2} \tilde{R}(u_1-u_2)^{\varepsilon'_1\varepsilon'_2}_{\varepsilon_1\varepsilon_2}\Psi^*_{\varepsilon'_1}(u_1)\Psi^*_{\varepsilon'_2}(u_2) = \Psi^*_{\varepsilon_2}(u_2)\Psi^*_{\varepsilon_1}(u_1), \tag{17}$$

$$\Psi_{\varepsilon_1}(u_1)\Phi_{\varepsilon_2}(u_2) = \tau(u_1-u_2)\Phi_{\varepsilon_2}(u_2)\Psi_{\varepsilon_1}(u_1), \tag{18}$$

$$\Psi_{\varepsilon_1}(u')\Psi^*_{\varepsilon_2}(u) = \frac{1}{\pi}\frac{\delta_{\varepsilon_1\varepsilon_2}}{u'-u} + O(1), \qquad \Psi_\varepsilon(u) = \Psi^*_{-\varepsilon}(u-1). \tag{19}$$

Here the $\tilde{R}(u)$ matrix is defined by the weights

$$\tilde{a}(u) = -a(u)|_{r\to r-1}, \quad \tilde{b}(u) = -b(u)|_{r\to r-1}, \quad \tilde{c}(u) = -c(u)|_{r\to r-1}, \quad \tilde{d}(u) = d(u)|_{r\to r-1} \tag{20}$$

and provides the S matrix of the eight-vertex model. This form of the S matrix was confirmed by Takebe by means of the Bethe ansatz [2].

The function $\tau(u)$ here is the same function as in the similar commutation relation (16) for the SOS model. It is not surprising, because we know that the spectra of transfer matrices of both models coincide.

How to calculate anything with these bosonic fields? Denote by Tr_* the trace over oscillator modes and by H^* the oscillator contribution to H_{mn}. Besides, let

$$\chi^* = \mathrm{Tr}_*(x^{4H^*}) = \frac{1}{(x^4;x^4)_\infty}.$$

Then, according to the Wick theorem,

$$\mathrm{Tr}_{\mathcal{F}_{mn}}(U_N(u_N)\dots U_1(u_1)) = \langle P_{mn}|U^0_N(u_N)\dots U^0_1(u_1)|P_{mn}\rangle\, \chi^* \prod_{i=1}^N c_i \prod_{i<j} g_{ij}(u_i-u_j)$$

with

$$\log c_i = \frac{1}{\chi^*}\,\mathrm{Tr}_*(\phi^+_i(0)\phi^-_i(0)x^{4H^*}),$$
$$\log g_{ij}(u) = \frac{1}{\chi^*}\,\mathrm{Tr}_*(\phi_i(0)\phi_j(u)x^{4H^*}), \qquad \phi_i(u) = \phi^+_i(u)+\phi^-_i(u).$$

The constants c_i and functions g_{ij} are expressed in terms of the infinite products $(z;p_1,\dots,p_k)_\infty$ defined above. The resulting form factors are expressed in terms of integrations of the products of these functions.

We know the vertex–face correspondence for the weights. Is it possible to relate somehow the vertex operator algebras? Surely, it is, but we need again a kind of 'physical reasoning'. You remember the relation

$$[m']\,\mathrm{Tr}_{\mathcal{H}^{(i)}}(x^{2H}) = \sum_{n\in 2\mathbb{Z}+m+i} \mathrm{Tr}_{\mathcal{H}_{mn}}\, x^{4H_{mn}}.$$

Analysis of the low temperature expansion really indicates the relation between ith ground state of the eight vertex model and the $n-m=i \pmod 2$ ground states of the SOS model. So, suppose that there exist the operators

$$T(u_0)_{mn} : \mathcal{H}_{mn}\to\mathcal{H}^{(i)}, \qquad T(u_0)^{mn} : \mathcal{H}^{(i)}\to\mathcal{H}_{mn}, \qquad i=n-m \pmod 2, \tag{21}$$

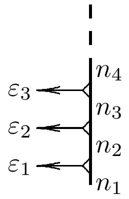
such that

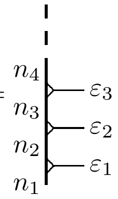
$$[m]'x^{2H^{(i)}} = \sum_{n\in 2\mathbb{Z}+m+i} T(u_0)_{mn} x^{4H_{mn}} T(u_0)^{mn} \tag{22}$$

and

$$\begin{aligned} \sum_n t_\varepsilon(u-u_0)_n^{n'} \Phi(u)_n^{n'} T(u_0)^{mn} &= T(u_0)^{mn'} \Phi_\varepsilon(u) \\ \sum_\varepsilon t^*_\varepsilon(u-u_0)^n_{n'} \Phi_\varepsilon(u) T(u_0)_{mn} &= T(u_0)_{mn'} \Phi(u)_n^{n'}, \end{aligned} \tag{23}$$

The operators $T(u_0)^{mn}$ and $T(u_0)_{mn}$ can be considered as a half of the $\tau(u_0)$ transfer matrix and its 'conjugate' $\tau^*(u_0)$ in the infinite volume limit:

$T(u_0)^{mn} =$ 

$T(u_0)_{mn} =$ 

How the operators $T(u_0)^{mn}$ and $T(u_0)_{mn}$ intertwine the type II operators? In the spirit of this algebraic approach, we can suggest that the commutation relation look like

$$\begin{aligned} \Psi^*_\varepsilon(u) T(u_0)_{mn} &= \sum_{m'} T(u_0)_{m'n} \Psi^*(u)_m^{m'} \tilde{t}^*_\varepsilon(u-u_0-\Delta u_0)^m_{m'}, \\ \Psi^*(u)_m^{m'} T(u_0)^{mn} &= \sum_\varepsilon T(u_0)^{m'n} \Psi^*_\varepsilon(u) \tilde{t}_\varepsilon(u-u_0-\Delta u_0)_m^{m'}, \end{aligned} \tag{24}$$

where Δu_0 is some shift and $\tilde{t}_\varepsilon(u)_m^{m'}$ are intertwining functions after the substitution $r \to r-1$ and erasing $e^{\pm \mathrm{i}\pi/4}$:

$$\begin{aligned} \tilde{t}_+(u)_n^{n'} &= \tilde{f}(u)\theta_3\left(\frac{(n'-n)u+n'}{2r'}; \mathrm{i}\frac{\pi}{2\epsilon r'}\right), \\ \tilde{t}_-(u)_n^{n'} &= \tilde{f}(u)\theta_4\left(\frac{(n'-n)u+n'}{2r'}; \mathrm{i}\frac{\pi}{2\epsilon r'}\right), \qquad r' = r-1. \end{aligned} \tag{25}$$

The normalization conditions fix the overall factors $f(u)$ and $\tilde{f}(u)$ to be solutions of the equations

$$\begin{aligned} [u]f(u)f(u-1) &= C \equiv \frac{[0]_4^2}{2\theta_3(0;\mathrm{i}\pi/2\epsilon r)\theta_4(0;\mathrm{i}\pi/2\epsilon r)}, \\ [u]'\tilde{f}(u)\tilde{f}(u-1) &= C' \equiv \frac{[0]_4'^2}{2\theta_3(0;\mathrm{i}\pi/2\epsilon r')\theta_4(0;\mathrm{i}\pi/2\epsilon r')}. \end{aligned}$$

The function $f(u)$ is not essential for the answers since the type I vertex operators Φ are always accompanied by the 'conjugates' Φ^*. But the particular form of $\tilde{f}(u)$ is essential and will be fixed on the basis of the bosonization procedure as well as the value of the shift Δu_0.

Consider any form factor from the eight-vertex model

$$\frac{1}{\chi^{(i)}} \mathrm{Tr}_{\mathcal{H}^{(i)}} (\Phi^*_{\varepsilon'_1}(u_1) \ldots \Phi^*_{\varepsilon'_k}(u_k) \Phi_{\varepsilon_k}(u_k) \ldots \Phi_{\varepsilon_1}(u_1) \Psi^*_{\alpha_l}(v_l) \ldots \Psi^*_{\alpha_1}(v_1) x^{2H^{(i)}})$$

We can represent the operator $x^{2H^{(i)}}$ according to (22) and push $T(u_0)_{mn}$ to the left and $T(u_0)^{mn}$ to the right by use of the intertwining relations (23) and (24). We obtain an infinite liner combination of the traces

$$\frac{[n]}{[m']'\chi^{(i)}} \mathrm{Tr}_{\mathcal{H}_{m'n}} (\Phi^*(u_1)^n_{n'_1} \ldots \Phi^*(u_k)^{n'_{k-1}}_{n''} \Phi(u_k)^{n''}_{n_{k-1}} \ldots \Phi(u_1)^{n_1}_{n'} \Lambda(u_0)^{m'n'}_{mn} \Psi(u_l)^m_{m_{l-1}} \ldots \Psi(u_1)^{m_1}_{m'} x^{4H_{m'n}})$$

with

$$\Lambda(u_0)_{mn}^{m'n'} = T(u_0)^{m'n'} T(u_0)_{mn} \; : \; \mathcal{H}_{mn} \to \mathcal{H}_{m'n'}. \tag{26}$$

All ingredients in this trace are known from the free field representation except the operators $\Lambda(u_0)_{mn}^{m'n'}$. In the next lecture we fix the bosonic form of these operators using the commutation relations of this operator with the vertex operators.

References

[1] O. Foda, K. Iohara, M. Jimbo, R. Kedem, T. Miwa, H. Yan, *Lett. Math. Phys.* **32** (1994) 259 [hep-th/9403094]

[2] T. Takebe, *J. Phys. A* **28** (1995) 6675; corrigendum **29** (1996) 1563 [q-alg/9504027]