

**Course Title (in English)** Methods of the theory of one-dimensional quantum systems

**Course Title (in Russian)** Методы теории одномерных квантовых систем

**Lead Instructor(s)** Lashkevich, Michael

**Status of this Syllabus** The syllabus is a final draft waiting for form approval

**Contact Person** Michael Lashkevich

**Contact Person's E-mail** lashkevi@landau.ac.ru

## 1. Annotation

### Course Description

In the framework of the course, quantum systems (field-theoretic and discrete) in one spacial dimension, and some their classical statistical mechanics counterparts are discussed. The scope of systems includes sine-Gordon and Thirring model,  $O(n)$  sigma model, Heisenberg chain and six-vertex model, Kondo problem. We consider several techniques to obtain exact results for these systems, including operator product expansions, boson-fermion correspondence, Yang-Baxter equation, different versions of Bethe Ansatz.

**Course Prerequisites** Students must be familiar with basics of quantum field theory and statistical mechanics.

## 2. Structure and Content

**Course Academic Level** Master-level course suitable for PhD students

**Number of ECTS credits** 6

Topic	Summary of Topic	Lectures (# of hours)	Seminars (# of hours)	Labs (# of hours)
Berezinsky-Kosterlitz-Thouless (BKT) transition and sine-Gordon quantum field theory	O(2) model and BKT transition. Its description by the sine-Gordon quantum field theory. Renormalization group for the BKT transition.	2	2	
O(N) sigma models	O(N) sigma models in classical and quantum case. 1/N expansion and quantum mass generation. Calculation of the approximate S matrix. Integrals of motion and exact S matrix factorization. Yang-Baxter equation and the exact S matrix.	2	2	
Thirring model	Operator product expansions and boson-fermion correspondence between the Thirring model and the sine-Gordon model. Dirac sea and its generalization to a system with interaction. Pseudovacuum and pseudoparticles. Solution of the Thirring model by means of the Bethe Ansatz.	2	2	
Heisenberg chain and vertex models	The XYZ Heisenberg chain. Solution of the XY Heisenberg chain via fermionization. Solution of XXZ chain by means of the coordinate Bethe Ansatz. Six-vertex model as a statistical mechanics of the XXZ chain. R matrix and Yang-Baxter equation. Solution of the six-vertex model (and, hence, of the XXZ chain) by means of the algebraic Bethe Ansatz.	3	3	
Kondo problem	Kondo problem and its reduction to a system of one-dimensional electrons. Spin subsystem, transfer matrix and algebraic Bethe Ansatz. Solving Bethe equations and T=0 properties of Kondo system. Remarks on how to solve Bethe equations for nonzero temperatures.	2	2	
Solving problems and final evaluation	During the course students are proposed to solve 38 problems. At least 40% of solved problems are necessary for any positive mark, and at least 80% for the excellent mark.		4	

### 3. Assignments

Assignment Type	Assignment Summary
Homework	Homework consists of 38 problems to solve (see above about final evaluation).

### 4. Grading

Type of Assessment	Graded	
Grade Structure	Activity Type	Activity weight, %
	Homework Assignments	100%

### Grading Scale

A:	80
B:	70
C:	60
D:	50
E:	40
F:	0

## 5. Basic Information

**Attendance Requirements** Mandatory with Exceptions

Maximum Number of Students	Maximum Number of Students	
	Overall:	10
	Per Group (for seminars and labs):	

**Course Stream** Science, Technology and Engineering (STE)

**Course Term (in context of Academic Year)** Term 5

**Course Delivery Frequency** Every year

Students of Which Programs do You Recommend to Consider this Course as an Elective?	Masters Programs	PhD Programs
	Mathematical and Theoretical Physics	Physics

**Course Tags** Math  
Physics

## 6. Textbooks and Internet Resources

Recommended Textbooks	ISBN-10 or ISBN-13
A.M.Tsvelik, Quantum Field Theory in Condensed Matter Physics	0521529808
V.E.Korepin, N.M.Bogoliubov, A.G.Izergin, Quantum Inverse Scattering Method and Correlation Functions	9780511628832

Web-resources	Description
<a href="http://homepages.itp.ac.ru/~lashkevi/lectures/2d-qft/">http://homepages.itp.ac.ru/~lashkevi/lectures/2d-qft/</a>	

## 7. Facilities

## 8. Learning Outcomes

### Knowledge

Some solvable systems, their indications and features

Boson-fermion correspondence

Yang-Baxter equation and its applications in physics

Bethe Ansatz and Bethe equations

### Skill

Basic skill for analyzing and solving various models

**Do you want to specify outcomes in another framework?**

Knowledge-Skill-Experience is good enough

9. Assessment Criteria

**Select Assignment 1 Type**

Homework

**Or Upload Example(s) of Assignment 1**

<https://ucarecdn.com/5b17311d-c56e-4df0-861a-d3a9d2ad02e5/>

10. Additional Notes